An Adaptive Partition Strategy of Galerkin Boundary Element Method for Capacitance Extraction

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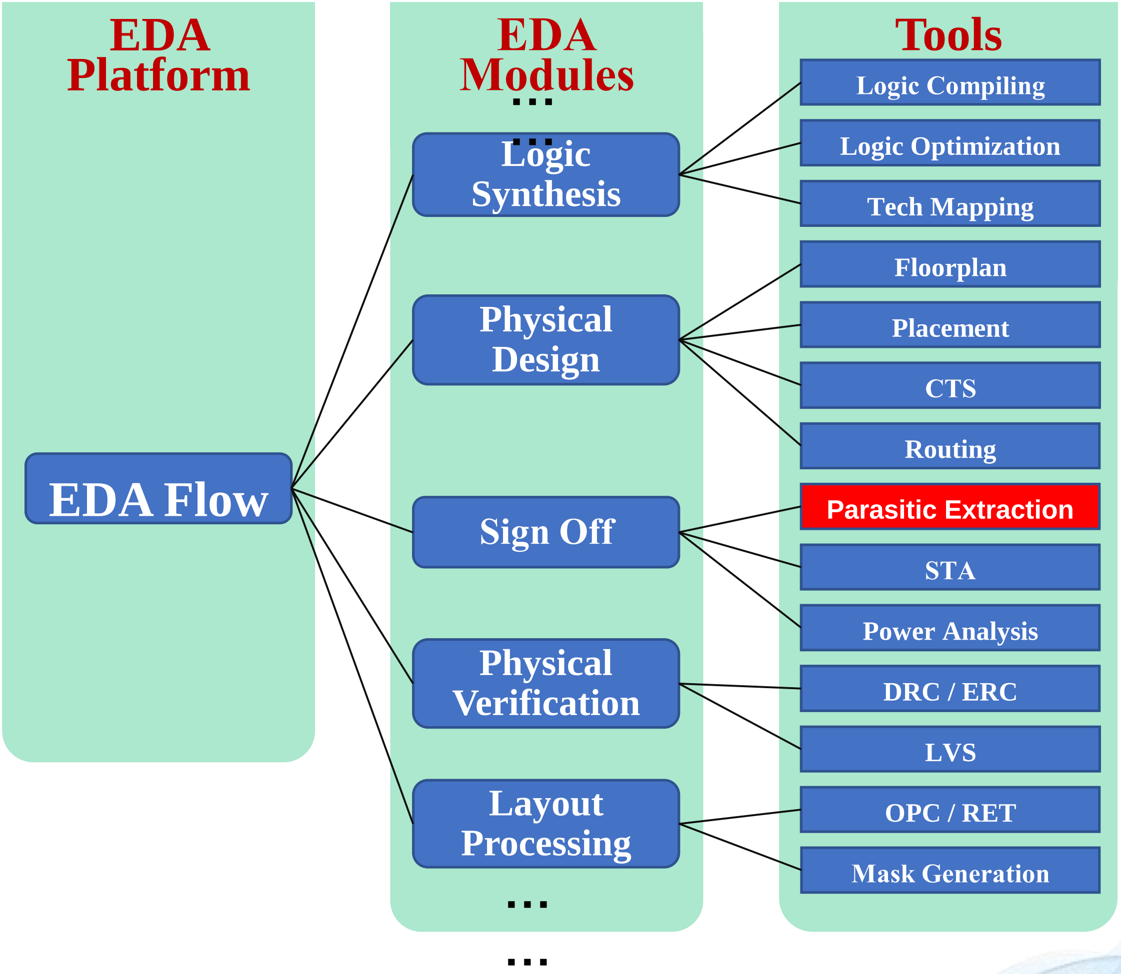
Capacitance Extraction

Problem:

The EDA flow includes logic synthesis, physical design, sign off and so on. Parasitic extraction is a part of sign off analysis. Parasitic extraction includes resistance extraction and capacitance extraction.

In this report, we are going to discuss capacitance extraction.



EDA Flow

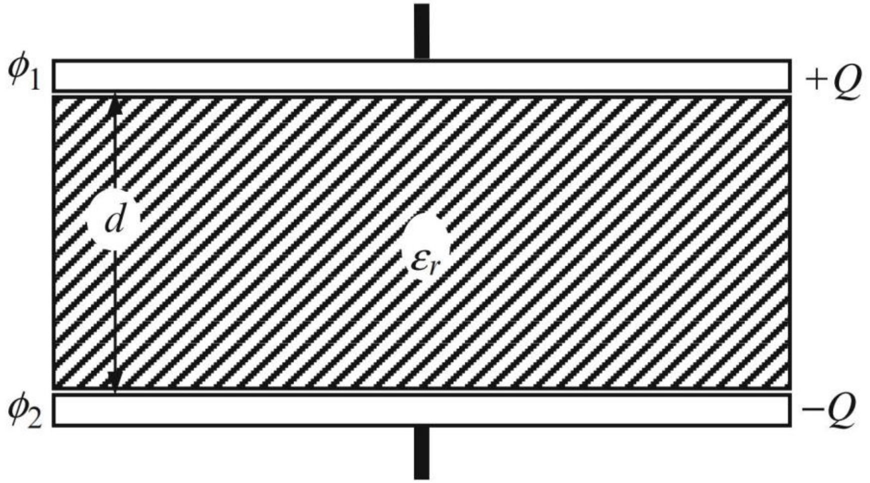
Capacitance Extraction

Capacitor:

The capacitor is a kind of circuit elements commonly used in electric or electronic equipment. It is usually composed of two conductors insulated from each other. When charged, the two surfaces of the conductors facing each other carry equal and opposite charges: Q and $-Q$, respectively. The electric potential on the two conductors are ϕ_1 and ϕ_2, respectively.

The capacitance of the capacitor is denoted by C:

$$C = \frac{Q}{\phi_1 - \phi_2}\ .$$



A parallel plate capacitor

Motivation of Capacitance Extraction:

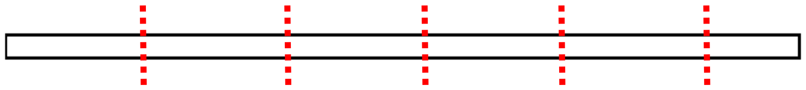
The motivation of capacitance extraction is to compute signal delay in integrated circuit.

For a signal wire in an integrated circuit, to compute its signal delay from one side to another side, a discretization is adopted to approximate the wire with an RC tree.

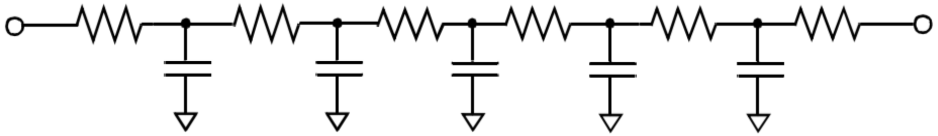
Let $X(t)$ be the voltage on all nodes. Then the voltage can be solved by following equation:

$$\dot{X}(t) = AX(t) + B,$$

where A and B are two matrices that depend on the resistance and capacitance of this RC tree.



(a) Trace of length L.



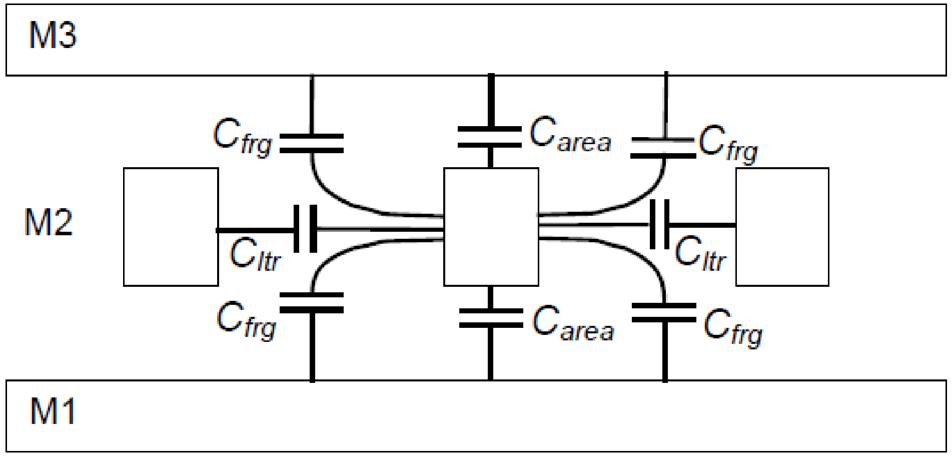
(b) Distributed RC tree.

Capacitance Extraction

Integrated Circuit:

For IC design, the requirement of fast and accurate capacitance extraction is becoming more and more urgent.

With the feature size of integrated circuit scaling down, the coupling capacitance of interconnect wires is making more and more significant impact on circuit performance.

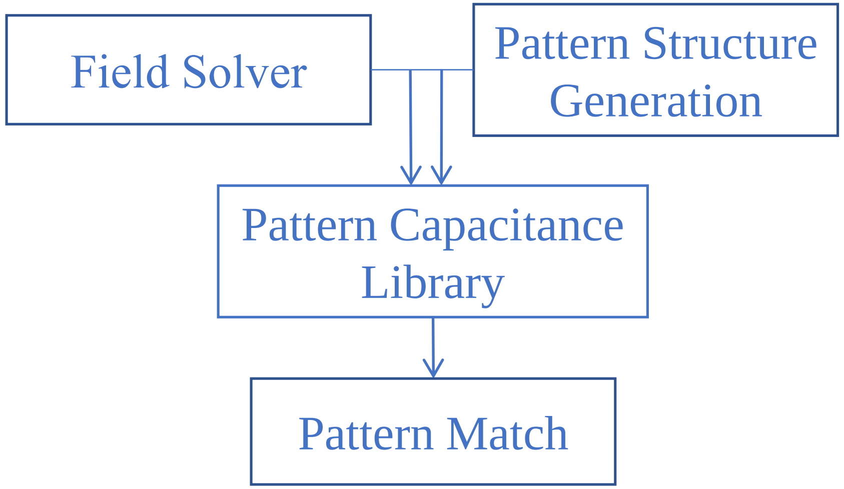


Coupling capacitance of interconnect wires

Capacitance Extraction

Two Key Parts:

To obtain a good trade-off between accuracy and efficiency. A complete capacitance extraction tool includes mainly two important parts that are field solver and pattern match.



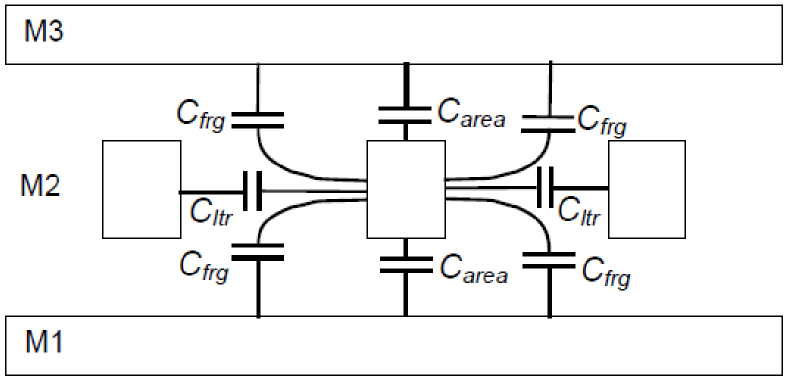
Field Solver:

Field solver is an accurate and general method for calculating the capacitance parameters through simulating the electrostatic field among conductors.

Pattern Match:

Pattern match is a procedure which use the pattern capacitance library as a lookup tables or a training dataset to train some empirical formulas.

The capacitance of a given conductor structure are obtained either to use some empirical formulas or to search from a lookup table (“pattern library”).



Field Solver

Sevaral Field Solver Methods:

There are mainly several capacitance extraction methods which are the finite difference method (FDM), the finite element method (FEM), the boundary element method (BEM) and the floating random walk (FRW) method.

Table 1: Features of different Field Solver.

–	FDM or FEM	BEM	FRW
Equation form	differential	integral	integral
Discretization	domain	boundary	–
Matrix	large and sparse	small and dense	–
Parallelism	bad	bad	good
Convergence rate	rapid	rapid	slow
Main error source	discretization	discretization	random
Adaptability to complex structure	good	bad	bad

Field Solver

Domain Discretization Method:

The FEM and the FDM are classified as the domain discretization method. It usually produces a sparse matrix with large order. In 3-D capacitance extraction, because the order of the matrix increase rapidly, the speed of this method is limited. However, the domain discretization method is well established, thus this method is used by some software.

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Field Solver

Floating Random Walk Method:

The floating random walk algorithm for capacitance extraction, presented as a 2-D version, was proposed in 1992.

The random walk method is advantageous in parallelism over the traditional methods. Thus, this method has attracted a lot of attention recently.

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Adaptability to complex structure	good	bad	bad

Field Solver

Boundary Element Method（BEM）：

The boundary element method only needs to discretize the boundary, thus the matrix order produced by the BEM is smaller than that produced by the FDM. However, the matrix obtained by the BEM is not sparse and a lot of time is spent on calculating the matrix elements.

Table 1: Features of different Field Solver.

–	FDM or FEM	BEM	FRW
Equation form	differential	integral	integral
Discretization	domain	boundary	–
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Adaptability to complex structure	good	bad	bad

[1] X. Cai, K. Nabors, and J. White. 1995. Efficient Galerkin techniques for multipole-accelerated capacitance extraction of 3-D structures with multiple dielectrics. In Proceedings of the Conference on Advanced Research in VLSI. 200–211.
[2] W. Chai, D. Jiao, and C.-K. Koh. 2009. A direct integral-equation solver of linear complexity for large-scale 3-D capacitance and impedance extraction. In Proceedings of Design Automation Conference. 752–757

Field Solver

Galerkin Boundary Element (GBEM) Method:

We focus on the matrix order reduction and study the boundary partition strategy.

1. For this purpose, we use some mathematical theorems and physical approximation to obtain an error analysis. The error analysis can provide a guidance of boundary partition strategy. Thus, the boundary partition strategy is rarely influenced subjectively.

2. Due to the fitness, this boundary partition strategy can largely reduce the number of boundary elements and ensure sufficient accuracy.

3. On the other hand, we will also propose our suggestion on the calculation of the matrix elements.

Field Solver

Problem Statement:

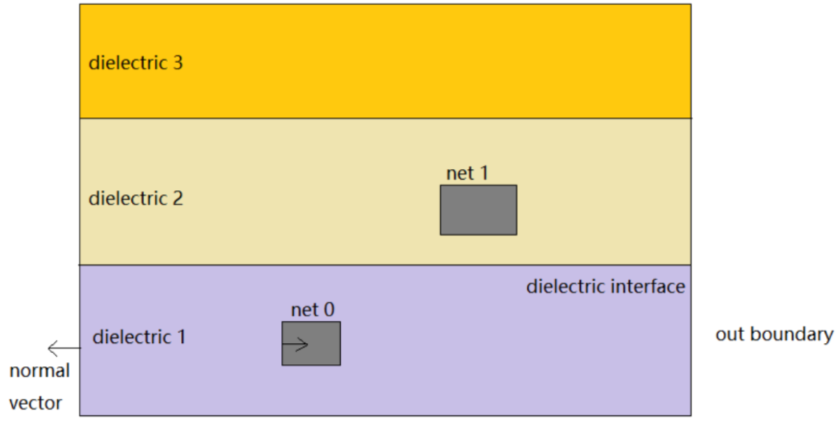


Figure 1: A cross section of 3-D capacitance extraction problem with multi-dielectrics

In each dielectric region Ω_a, the electric potential u_a satisfies the Laplace equation:

$$\begin{cases} \nabla^2 u_a = 0 \text{ in } \Omega_a \\ u_a = 1 \text{ on the main net} \\ u_a = 0 \text{ on other nets and the Dirichlet boundary} \\ \dfrac{\partial u_a}{\partial n} = 0 \text{ on the Neumann boundary.} \end{cases}$$

Let q_a and q_b be the normal electric field intensity on the boundaries of Ω_a and Ω_b respectively. On the dielectric interface Γ_{ab} of two dielectric regions Ω_a and Ω_b, the compatibility equation holds:

$$\varepsilon_a q_a = -\varepsilon_b q_b$$

$$u_a = u_b.$$

Using Green's identity, we have following equation

$$\sigma(x) u_a(x) + \int_{\partial\Omega_a} q^*(x,y) u_a(y) dy = \int_{\partial\Omega_a} u^*(x,y) q_a(y) dy,$$

where

$$q^*(x,y) = \frac{\langle x-y, n_y \rangle}{4\pi |x-y|^3} \quad , \quad u^*(x,y) = \frac{1}{4\pi |x-y|}$$

and $\sigma(x)$ satisfies $\sigma(x) = \frac{1}{2}$ for almost all $x \in \partial\Omega_a$.

Galerkin Method

Single Dielectric Case:

First, we discuss the single dielectric case with the Dirichlet boundary condition:

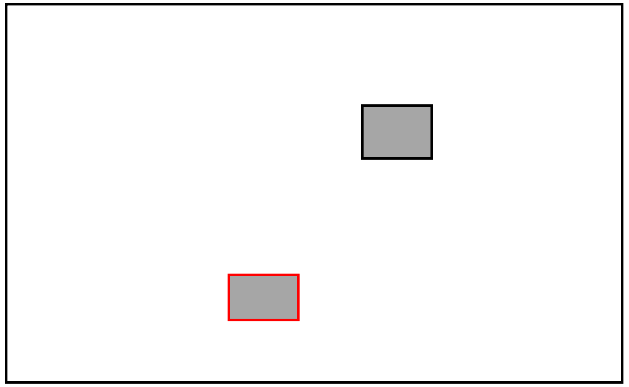
$$u = 0 \text{ on the dielectric boundary .}$$

Let ∂mcd represent the boundary of the main conductor(mcd). Then, the basic theory of electromagnetism tells us that

$$\int_{\partial\Omega} u^*(x,y)q(y)dy = f_0(x),$$

where

$$u^*(x,y) = \frac{1}{4\pi|x-y|} \qquad f_0(x) = \begin{cases} 1 & \text{if x is on } \partial mcd \\ 0 & \text{otherwise.} \end{cases}$$



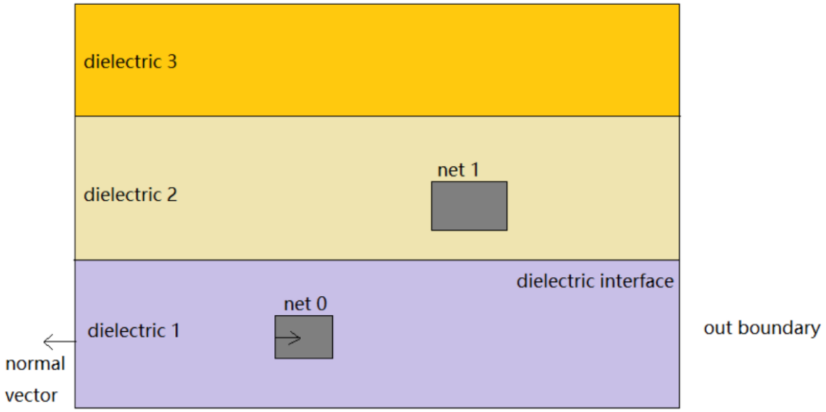


Figure 1: A cross section of 3-D capacitance extraction problem with multi-dielectrics

Galerkin Method

Single Dielectric Case:

To solve q, we need to partition the boundary to finite pieces. We suppose $\partial\Omega = \bigcup_j I_j$ and use q_j to approximate $q(x)$ when $x \in I_j$, then we get following equation:

$$\sum_j \left(\int_{I_j} u^*(x,y)dy \right) q_j = f_0(x).$$

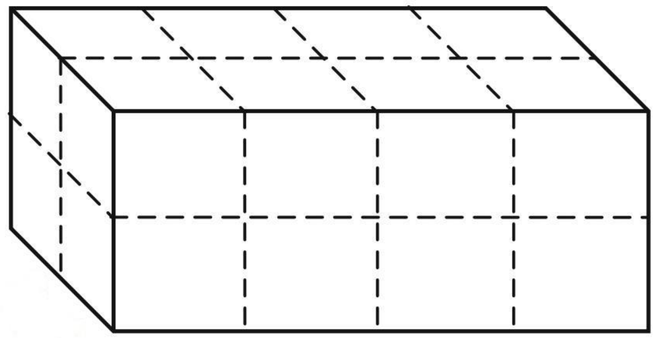
Integrating both sides over I_i, we get

$$\sum_j \left(\int_{I_i} \int_{I_j} u^*(x,y)dydx \right) q_j = \int_{I_i} f_0(x)dx. \qquad (3)$$

This is the Garlerkin method discussed in [2]. Once we have solved this equation, the electric charge on the boundary ∂cd_k of the k-th conductor (cd_k) is given by

$$Q_k' = \sum_{I_j \subset \partial cd_k} \varepsilon q_j |I_j|, \qquad (4)$$

where ε is the dielectric constant. Let



partition of a conductor boundary

$$\int_{\partial\Omega} u^*(x,y)q(y)dy = f_0(x),$$

Galerkin Method

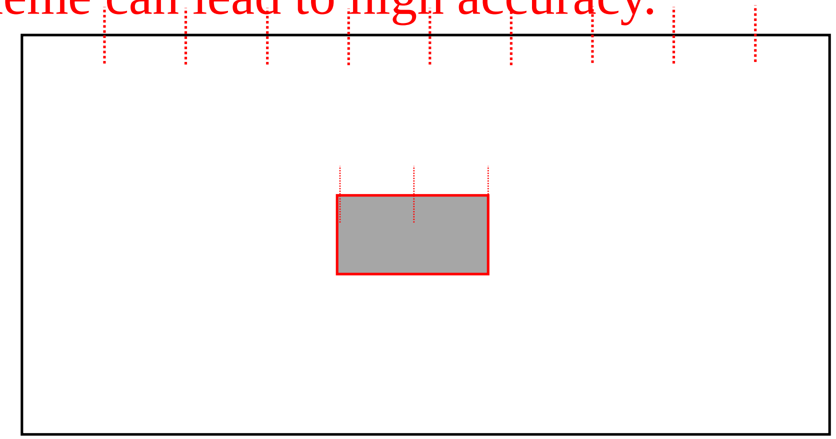
Boundary Partition Strategy:

$$A = \left\{ \frac{1}{|I_i|} \frac{1}{|I_j|} \int_{I_i} \int_{I_j} u^*(x, y) dy dx \right\}_{i,j}.$$

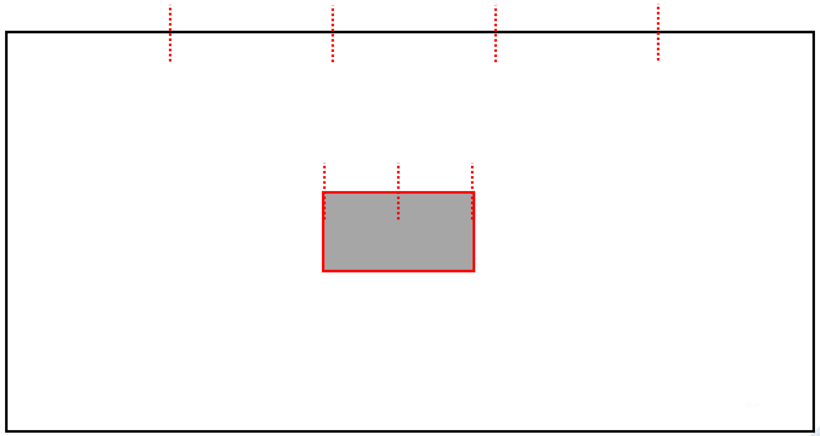
The order of the Matrix A is equal to the number of the boundary elements. Thus we want to reduce the number of boundary elements.

An intuitive way is to use nonuniform partition. For example, the size of the boundary elements on the main conduct should be small and the size of the boundary elements on the dielectric boundary can be large.

Nonuniform partition lead to less boundary elements. We need theorems to ensure that this partition scheme can lead to high accuracy.



uniform partition



nonuniform partition

Galerkin Method

Error Estimation:

Proposition 1. Let Q_k be the electric charge on the conductor k. Let Q_k' be the electric charge on the conductor k obtained by Galerkin method. The projection q_m is given by

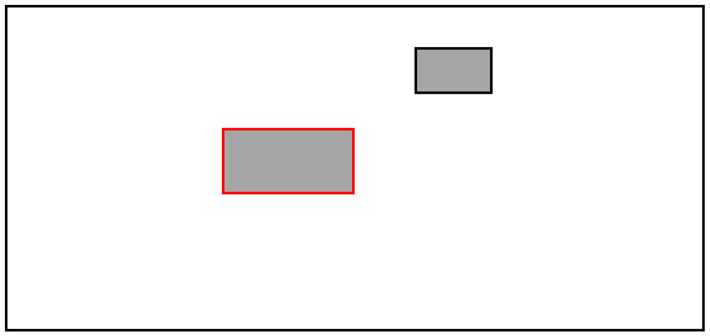
$$P_m q = \sum_j \frac{1}{|I_j|} \int_{I_j} q(x)dx\chi_{I_j},$$

then there is a constant C such that

$$|Q_k - Q_k'| \le C\|q - P_m q\|_{L^2(\partial\Omega)}.$$

If the tangential derivative of q on the boundary exists, we further have

$$|Q_k - Q_k'| \le C \sup_j \left(|I_j| \sup_{x\in I_j} \frac{\partial q}{\partial \tau}(x)\right).$$



Since the capacitance of the main conductor is given by $C_1 = Q_1$ and the coupling capacitance between the main conductor and the conductor k is given by $C_k = -Q_k$, we can obtain the error estimation of each capacitance.

Galerkin Method

Partition Strategy:

If we want the error to be less that c, we need to partition the boundary such that

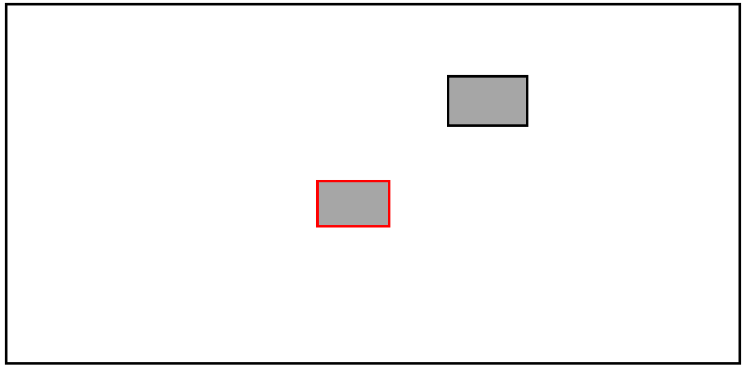
$$|I_j| \sup_{x\in I_j} |\frac{\partial q}{\partial \tau}(x)| \leq c$$

$$|I_j| \leq \frac{c}{\sup_{x\in I_j} |\frac{\partial q}{\partial \tau}(x)|}.$$

If x is far away from the main conductor, then we can regard the main conductor as a point charge. Thus we have

$$|\frac{\partial q}{\partial \tau}(x)| \leq \frac{C}{|x - x_0|^3} \text{ for some constant } C.$$

Let d_{I_j} be the distance from I_j to the main conductor. Form formulas above, we should partition the boundary such that $|I_j|$ is propotional to $(d_{I_j})^3$ when d_{I_j} is large.



$$|Q_k - Q_k'| \leq C \sup_j \left(|I_j| \sup_{x\in I_j} \frac{\partial q}{\partial \tau}(x)\right).$$

Galerkin Method

Partition Strategy:

Now, we propose our strategy of partition. Let p_1, p_2, p_3 are parameters which can be chosen for different cases.

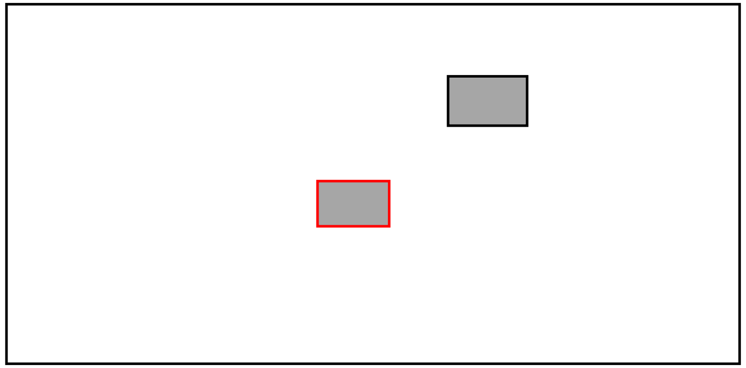
If the distance between the boundary and the main conductor is less than p_1, then we partition it to rectangles $\{I_j\}$ such that

$$|I_j| \leq p_2.$$

If the distance of the boundary is larger than p_1, then we partition it to rectangles $\{I_j\}$ such that

$$|I_j| \leq p_3 * p_2 * (d_{I_j}/p_1)^3,$$

where d_{I_j} is the distance between I_j and the main conductor. Due to the electric field shielding effect, if the outer normal vector of the boundary points to the main conductor, we set $p_3 = 1$ else we set p_3 to be larger than 1.

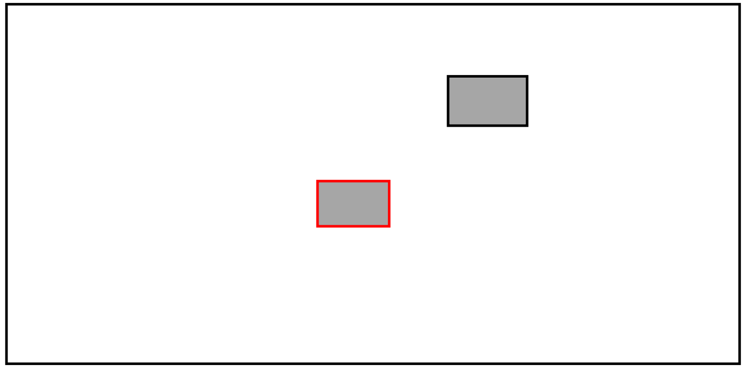


Galerkin Method

Multi-Dielectric Case:

For the multi-dielectric case, we have

$$\sigma(x)u_a(x) + \int_{\partial\Omega_a} q^*(x,y)u_a(y)dy = \int_{\partial\Omega_a} u^*(x,y)q_a(y)dy,$$



hold for x in region Ω_a. We partition the boundary to be the union of I_j. Taking integral over I_j on both side and approximating $q_a(y)$ and $u_a(y)$ on each element with constant, we then obtain

$$\frac{1}{2}\int_{I_i} u_a dx + \sum_k \left(\int_{I_i}\int_{I_k} q^*(x,y)dydx\right) u_{a,k}$$

$$= \sum_j \left(\int_{I_i}\int_{I_j} u^*(x,y)dydx\right) q_{a,j}.$$

Fast Calculation of Matrix Elements

Numerical Integral:

The Galerkin method brings us both a benefit and a trouble. The benefit is that the number of boundary elements is reduced. The trouble is that the matrix elements are complicated. Because most of the run time of any kind of boundary element methods is spent on computing matrix elements. It is crucial to compute matrix elements as fast as we can.

The matrix elements include two kind of integrals

$$A_{i,j} = \int_{I_i}\int_{I_j} u^*(x,y)dydx \quad \text{and} \quad Q_{i,j} = \int_{I_i}\int_{I_j} q^*(x,y)dydx.$$

A fast algorithm of the calculation of $A_{i,j}$ is given by [1].

[1]. Jitesh Jain, Cheng-Kok Koh, and Venkataramanan Balakrishnan. 2006. Exact and Numerically Stable Closed-Form Expressions for Potential Coefficients of Rectangular Conductors. IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: EXPRESS BRIEFS 53, 6 (JUNE 2006), 200–211.

Fast Calculation of Matrix Elements

Numerical Integral:

To handle the integral, we need some notations. If $f(x)$ is a function, we denote

$$D_x(x_u, x_d)f(x) = f(x_u) - f(x_d)$$

and

$$\begin{aligned} &D_{x_1,\cdots,x_n}(x_{1u}, x_{1d}, \cdots, x_{nu}, x_{nd}) \\ =&D_{x_1}(x_{1u}, x_{1d})D_{x_2,\cdots,x_n}(x_{2u}, x_{2d}, \cdots, x_{nu}, x_{nd}). \end{aligned}$$

Let $\vec{n}(I_i)$ and $\vec{n}(I_j)$ be the normal vector of I_i and I_j respectively. We are going to consider several situations by discussing the directions of $\vec{n}(I_i)$ and $\vec{n}(I_j)$.

Fast Calculation of Matrix Elements

Numerical Integral:

If $\vec{n}(I_i)$ is parallel to $\vec{n}(I_j)$, we suppose I_i and I_j satisfies

$$I_i = \{(x_1, x_2, x_3) : x_{1d} \le x_1 \le x_{1u}, x_{2d} \le x_2 \le x_{2u}\}$$
$$I_j = \{(y_1, y_2, y_3) : y_{1d} \le y_1 \le y_{1u}, y_{2d} \le y_2 \le y_{2u}\}.$$

If $b = x3 - y3 = 0$, then by the formular of $q^*(x, y)$ we have $Q_{i,j} = 0$.

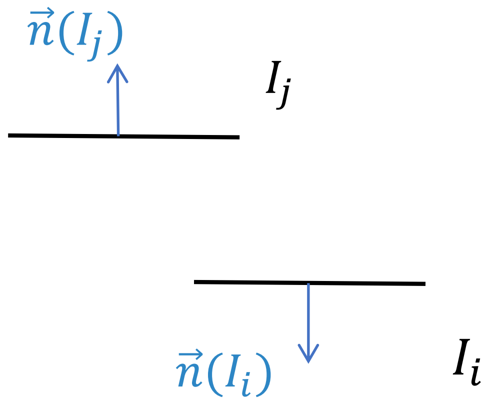
If $b = x3 - y3 \ne 0$, we have

$$Q_{i,j} = \int_{x_{1d}}^{x_{1u}} \int_{x_{2d}}^{x_{2u}} \int_{y_{1d}}^{y_{1u}} \int_{y_{2d}}^{y_{2u}} \frac{\langle x - y, n_y \rangle}{4\pi |x - y|^3} dy_2 dy_1 dx_2 dx_1$$
$$= D_{x_1, y_1}(x_{1u}, x_{1d}, -y_{1u}, -y_{2d}) \int_{x_{2d}}^{x_{2u}} \int_{-y_{2d}}^{-y_{2u}}$$
$$\frac{1}{4\pi} \frac{b\sqrt{(x_2 + y_2)^2 + (x_1 + y_1)^2 + b^2}}{(x_2 + y_2)^2 + b^2} dx_2 dy_2.$$

For any function G, we have

$$\int_{x_d}^{x_u} \int_{y_d}^{y_u} G(x + y) dy dx \left(\text{Let } \delta x = x_u - x_d, \delta y = y_u - y_d.\right)$$
$$= \int_0^{\delta y} u G(u + x_d + y_d) du + \int_{\delta x}^{\delta y} \delta y G(u + x_d + y_d) du \qquad (8)$$
$$+ \int_{\delta x}^{\delta y + \delta x} (\delta y + \delta x - u) G(u + x_d + y_d) du.$$

using (8), we can calculate $Q_{i,j}$.



Fast Calculation of Matrix Elements

Numerical Integral:

If $\vec{n}(I_i)$ is orthogonal to $\vec{n}(I_j)$, we suppose I_i and I_j satisfies

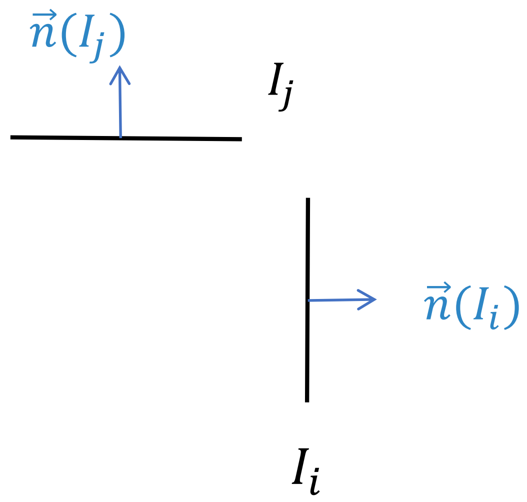
$$I_i = \{(x_1, x_2, x_3) : x_{1d} \le x_1 \le x_{1u}, x_{3d} \le x_3 \le x_{3u}\}$$

$$I_j = \{(y_1, y_2, y_3) : y_{1d} \le y_1 \le y_{1u}, y_{2d} \le y_2 \le y_{2u}\}.$$

Then

$$Q_{i,j} = D_{x_3,x_1,y_1}(\tilde{x}_{3u}, \tilde{x}_{3d}, x_{1u}, x_{1d}, -y_{1u}, -y_{1d}) \int_{\tilde{y}_{2d}}^{\tilde{y}_{2u}} (x_1 + y_1) \ln\left[(x_1 + y_1) + \sqrt{(x_1 + y_1)^2 + y_2^2 + x_3^2}\right] - \sqrt{(x_1 + y_1)^2 + y_2^2 + x_3^2} dy_2.$$

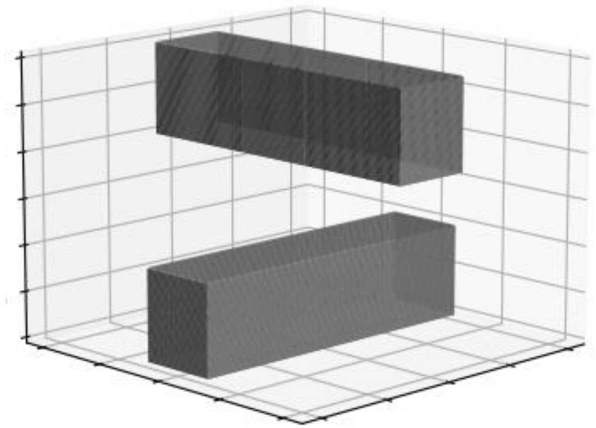
Thus, we can calculate $Q_{i,j}$ by using numerical integrals.



We used the Romberg quadrature formula to calculate each numerical integral in our program.

Experimental Result

We implemented our Galerkin projection boundary element method (GBEM) in C++ programming language, and performed all the experiments on Linux workstation with Intel(R) Xeon(R) Gold 5218 server with CPU at 2.3 GHz.



Single Dielectric Case:

We use the test case in [1]. We provide the capacitance obtained using different methods in [1] carried out on a Sun Ultra Enterprise 450 server. GBEM needs 74 boundary elements to obtain similar results. The errors are less that 5%.

Table 1: Capacitance calculated with different methods (in unit of 10^{-18} F).

Method	C_{11}	Error1(%)	C_{22}	Error2(%)	Time(s)	Mem(MB)
Raphael	232.2	0	181.5	0	–	–
GIMEI	230	0.94	180.6	0.5	2.8	3.5
FastCap	226	2.7	176	3	24.37	22
QMM	221	4.86	176.4	2.81	4.98	5.1
GBEM	222	4.4	178	1.93	0.06	7.19

[1]. W. Yu and X. Wang. 2014. Advanced Field-Solver Techniques for RC Extraction. Springer, New York

Experimental Result

Multiple Dielectric Case:

We provide the capacitance obtain using different methods carried out on a Sun Ultra Enterprise 450 server. "Panels" and "Cap." are the number of boundary elements and the capacitance.

For the QMM method, the number of boundary elements of a conductor ranges from 2277 to 2575 for capacitance computation. However, from Table 4, we can see that the number of boundary elements of a conductor in our GBEM only ranges from 217 to 316.

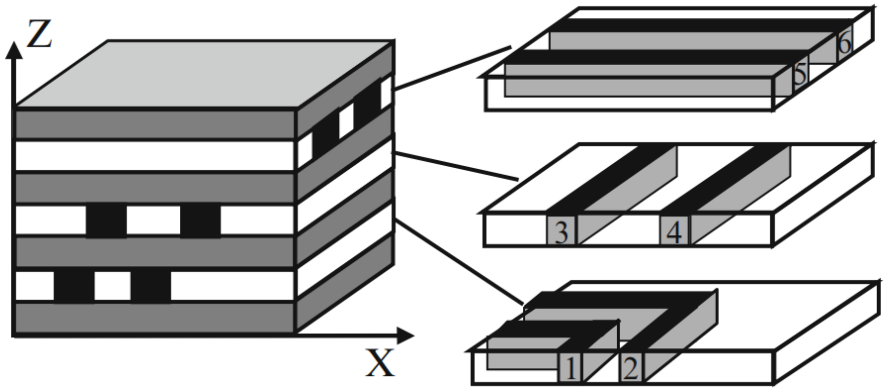


Table 4: Panels, times and memory for computing each capacitance (in unit of 10^{-15} F).

–	C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	Avg.
Panels	217	316	312	314	297	306	293
Time(s)	0.251	0.422	0.394	0.402	0.375	0.371	0.37
Mem(MB)	8.13	8.375	8.38	8.47	8.17	8.16	8.28
Cap.	0.691	1.306	1.598	1.542	2.544	2.539	

Table 2: The digonal entries of the capacitance matrices calculated with different methods (in unit of 10^{-15} F).

Method	C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	Time(s)	Mem(MB)
ODDM	0.68	1.29	1.57	1.52	2.54	2.54	122	2.7
QMM	0.682	1.31	1.6	1.54	2.53	2.53	58.4	3.80
GBEM	0.691	1.306	1.598	1.542	2.544	2.539	2.2	8.47

[1]. W. Yu and X. Wang. 2014. Advanced Field-Solver Techniques for RC Extraction. Springer, New York

Experimental Result

Multiple Dielectric Case:

Recently, there are a lot of progress on the floating random walk (FRW) solver for capacitance. One of efficient and effectiveness algorithms is RWCap(R) in [1].

The RWCap are carried out on a Linux server with Intel Xeon E5620 8-core CPU of 2.40 GHz. Our test cases are the typical 180-nm and 45-nm technology available from [1].

Table 3: Comparison of RWCap(R) and GBEM (in unit of 10^{-16} F).

	RWCap(R)				GBEM						
Case	C_{self}	C_{cl}	Time(s)	Walks	C_{self}	Dis. (%)	C_{cl}	Dis.(%)	Time(s)	Speedup	Panels
1	18.4	6.41	4.82	61k	18.7	1.6	6.28	2.03	3.48	1.39	1920
2	19.51	5.47	1.52	56k	19.5	0.05	5.5	0.55	2.61	0.58	1624
3	7.28	2.69	2.10	46k	7.13	2.06	2.65	1.49	1.77	1.19	1624
4	3.65	1.67	4.93	51k	3.62	0.82	1.63	2.4	0.538	9.16	1066
5	3.95	1.36	1.47	44k	3.92	0.76	1.40	2.94	1.89	0.78	1922
6	1.44	0.517	3.31	51k	1.46	1.39	0.494	4.45	0.674	4.91	1270

The experimental results in Table 3 show that our GBEM speed up runtime about 3.00× than RWCap(R).

[1]. W. Yu, H. Zhuang, C. Zhang, G. Hu, and Z. Liu. 2013. RWCap: A floating random walk solver for 3-D capacitance extraction of very-large-scale integration interconnects. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems 32, 3 (2013), 353–366

Thank You